JOURNAL OF MATERIALS SCIENCE 7 (1972) 861-869

Longitudinal tensile failure of unidirectional fibrous composites

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This paper presents a theoretical treatment of the tensile strength of a unidirectional fibrous composite, subjected to a tensile load in the fibre direction. The fibres are treated as having a statistical strength distribution which results in fibre failure prior to composite failure. The failure geometry of the model is similar to the observed geometry of fractured glass/epoxy and glass/polyester composites. Failure criterion is established and the strength is shown to decrease as the length of the specimen is increased. This size effect is very small.

1. Introduction

Strength of unidirectional fibrous composites when a tensile load is applied in the fibre direction has been studied by several investigators. The simplest approach is to assume that the failure strength of a given fibre population is a unique quantity. Thus, when all fibres in a composite specimen have the same tension, the specimen fails when the failure strain of the fibres is reached [1]. This approach is acceptable for ductile fibres, having essentially a unique failure strain. A comprehensive review of the literature dealing with longitudinal strength of fibrous composites has been presented by Kelly and Davies [2] with principal attention toward metal fibres embedded in a metal matrix.

Most fibres used in high strength and high stiffness composites are brittle, and their strength can generally only be characterized statistically due to distribution of flaws and imperfections in the brittle fibres. Parrott [3] included the statistical nature of fibre strength in the analysis of a fibrous composite. In his model, failure occurs when the number of fibre fractures caused by the increasing applied load increases to such a level that the unbroken segments of fibres are too short and the load cannot be transmitted to them because the matrix shear strength has been exceeded. Rosen [4] presented a theoretical and experimental treatment of the failure of fibrous composites. He treated the fibres as having a statistical distribution of flaws or imperfections that result in fibre fracture under an applied load even before total failure of the specimen. The load carried previously by a broken fibre is assumed to be distributed equally among all the unbroken fibres in the same cross section where the break is. Composite failure occurs when the weakest cross section cannot sustain the applied load. The theoretical prediction of strength given by Rosen is generally found to be higher than experimental values. Zweben [5] studied the influence of load concentrations caused by fibre breaks on the strength of two-dimensional composites, using the geometrical model of Rosen. It was shown that load concentrations had a significant effect on strength.

The failure model introduced by Rosen [4] assumes statistical accumulation of fibre fractures with increasing load until a sufficient number of fractures occur at some cross sectional region of the composite, resulting in composite failure. The fracture process, thus, takes place at a single cross section of the specimen. Some composite materials do in fact fail in such a manner (e.g., some carbon/epoxy composites). Experimental results with glass reinforced plastics (e.g, epoxy or polyester), however, show very complicated fracture surfaces (Fig. 1). The test specimen does not fail at a single cross section. Instead, bundles of fibres, grouped together, seem to fracture at different locations along the

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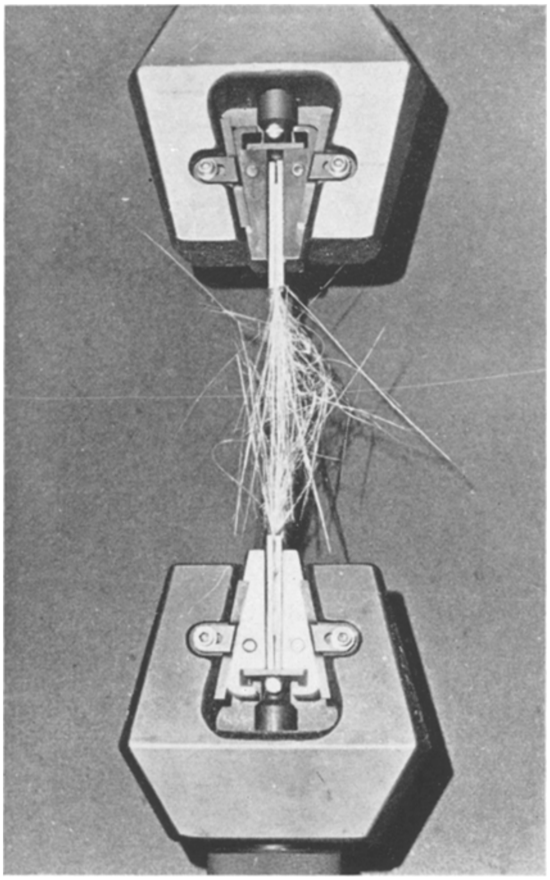

Figure 1 Typical failure of glass/polymer composite material.

specimen, followed by new fractured surfaces running parallel to the fibres. The final shape of the fractured specimen is shown in Fig. 1. It was therefore decided to propose a new failure model that agreed with the observed failure geometry. The proposed model is based on the cumulative weakening model of Rosen [4] but it is not assumed to fail in a single cross section.

Size effect in brittle materials has been studied previously, and in particular the effect of length on the strength of glass fibres has been investigated by Metcalfe and Schmitz [6]. It was found that strength of glass fibres exhibits a significant size effect. When glass fibres are used in a composite material the size effect is drastically reduced since failure of single filaments does not cause immediate failure of the composite. Investigation of size effect in unidirectional glass reinforced composites is rather difficult to conduct since the size effect is quite small. Some experimental results have been reported, however, showing a slight reduction in strength for longer specimens [7]. The results of the present analysis show that a small decrease in strength is expected when the size of the specimen is increased by a few orders of magnitude.

2. Description of model

The model proposed in this paper is based on the cumulative weakening model introduced by Rosen [4]. Rosen's model consists of parallel fibres in a homogeneous matrix subjected to a tensile load in the fibre direction. The fibres are treated as having a statistical distribution of flaws or imperfections which result in fibre failure at various stress levels. When a tensile load is applied to the specimen some fibres fracture at points of imperfection. The portion of a broken fibre near a break is not fully effective as a load carrying element. The model is considered to be composed of a series of layers perpendicular to the fibres as shown in Fig. 2. The thickness of each

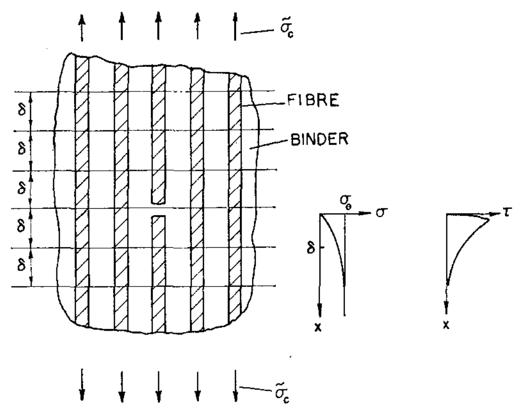

Figure 2 Tensile failure model of fibrous composite material (B. W. Rosen).

layer, δ, depends on the constituents' moduli and volume fraction of the fibres in the composite. It is a measure of the distance from a broken end of a fibre which is necessary to build the stress in the broken fibre to its normal value, by way of shear stresses in the matrix. The segment of a fibre within a layer may be considered as a link in the chain that constitutes the fibre. Each layer is a bundle of such links, and the composite is a series of such bundles.

When some fibres fracture, Rosen [4] assumes that the load is distributed uniformly among the unbroken fibres in each layer. Increasing load produces an increasing number of fibre fractures. Composite failure occurs when the remaining unbroken fibres at the weakest layer are unable to sustain the applied load. The result of this analysis for fibres characterized by a strength distribution of the Weibull type [8] is the definition of a statistical mode of the composite strength σ_c^*

$$\sigma_c^* = (\alpha\ \delta\ \beta \mathrm{e})^{-1/\beta} \qquad (1)$$

where α and β are constants describing fibre strength, δ is the thickness of a layer (or ineffective length) and e is the base of natural logarithms.

When a unidirectional fibrous composite (glass/epoxy) is subjected to a slow strain rate tensile test, in the fibre direction, it is observed that as the applied load approaches the value of the composite's strength, some groups of fibres (with the matrix material holding them together) break away from the specimen. (These groups of fibres are termed "strands" in this report.) The separation of a strand from the specimen occurs when the fibres contained in the strand break at some cross section, followed by a fracture of the matrix material (or the interface) surrounding the strand. The fracture of the surrounding matrix runs parallel to the fibres, from the broken cross section to the specimen's ends, since the matrix material cannot transfer the load carried previously by the fibres in the strand. The process of fracture and separation of such strands continues gradually as the stress in the remaining unbroken portion of the specimen increases. Similar process takes place when the rate of extension is high or when a dead load test is performed. Under those conditions, separation of strands occurs at a very high rate, following the fast increase in the value of the stress.

The failure process described heie has been observed in all tensile tests of glass/epoxy and glass/polyester unidirectonal composites [9]. The model that is proposed for such failure processes is composed of many strands in parallel which may fracture at different cross sections. When a strand fractures, the load is assumed to be distributed equally among the unbroken strands. Composite failure is supposed to occur when the remaining unbroken strands are unable to sustain the applied load.

The number of fibres in each of the strands depends on the mechanical properties of the fibres and the matrix material, the volume fraction of the fibres and their distribution in the matrix. A proposed method for predicting the number of fibres in a strand is given in the appendix.

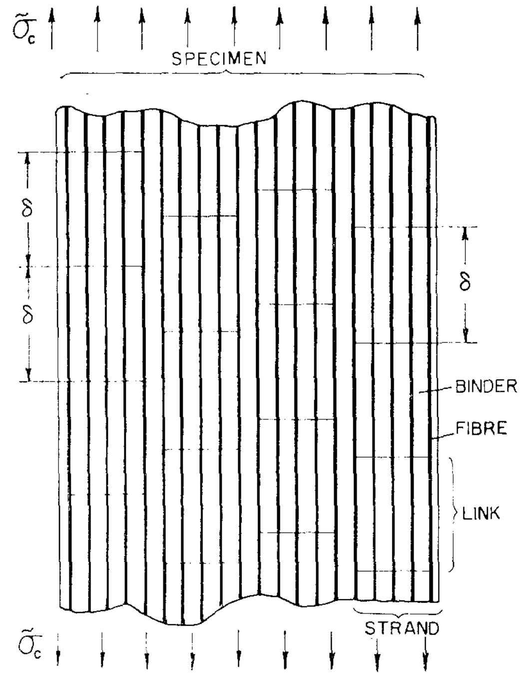

Figure 3 New tensile failure model of glass/polymer composite material.

A specimen that fails according to the mechanism described here is represented by the model shown in Fig. 3. The specimen is assumed to be composed of a series of strands connected in parallel, all having the same size. The length of a strand is equal to the specimen's length and each one of the strands is divided into layers of length δ (the ineffective length). The analysis of a strand is identical to the analysis of Rosen's model [4]. Once this analysis has been performed and the statistical strength characteristics of the strands are known, the analysis of the new model continues by considering the specimen to be composed of a bundle of such strands and using the statistical theory developed by Daniels [10] for calculating strength of bundles.

3. Statistical analysis of a strand

As mentioned earlier, the analysis of a strand whose length is equal to the specimen's length, is identical to the analysis of Rosen's model [4] and is presented here shortly for further use in the analysis of the specimen. The strength of a fibre link of length δ is characterized by a distribution function $f(\sigma)$, and the associated cumulative distribution function $F(\sigma)$, where

$$F(\sigma) = \int_0^\sigma f(\sigma)\, \mathrm{d}\sigma \qquad (2)$$

and σ is the fibre stress. When many fibre links are connected in parallel, thus forming a bundle, it has been shown by Daniels [10] that the average fibre stress at bundle failure, σ_B, approaches a normal distribution. The relative number of unbroken fibres at fibre stress σ is $1 - F(\sigma)$, and the applied stress, $\tilde{\sigma}_B$ is related to the fibre stress σ by:

$$\tilde{\sigma}_B = \sigma[1 - F(\sigma)] \tag{3}$$

The maximum fibre stress at bundle failure is denoted by σ_f and is found by maximizing $\tilde{\sigma}_B$ in Equation 3. The strength of a bundle is characterized by the normal distribution function

$$\omega(\sigma_B) = \frac{1}{\psi_B\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{\sigma_B - \bar{\sigma}_B}{\psi_B}\right)^2\right] \tag{4}$$

and the associated cumulative distribution function Ω

$$\Omega(\sigma_B) = \int_0^{\sigma_B} \omega(\sigma_B)\, d\sigma_B \tag{5}$$

with expectation

$$\bar{\sigma}_B = \sigma_f[1 - F(\sigma_f)] \tag{6}$$

and standard deviation

$$\psi_B = \sigma_f\left\{\frac{F(\sigma_f)}{N}[1 - F(\sigma_f)]\right\}^{\frac{1}{2}} \tag{7}$$

where N is the number of fibres in a bundle. The strand is treated as a chain composed of n links, where each link is a bundle of N fibres, characterized by Equations 3 to 7. The strength of such a chain is defined by the distribution function $\lambda(\sigma_s)$,

$$\lambda(\sigma_s) = n\,\omega(\sigma_s)\,[1 - \Omega(\sigma_s)]^{n-1} \tag{8}$$

where σ_s is the average fibre stress at strand failure. The associated cumulative distribution function is given by,

$$\Lambda(\sigma_s) = \int_0^{\sigma_s} \lambda(\sigma_s)\, d\sigma_s \tag{9}$$

4. Statistical analysis of a specimen

The basic element of Rosen's model [4] is the fibre link of length δ, the ineffective length. The reason for defining the ineffective length in his analysis is that when many fibres are connected in parallel and a fibre breaks, it becomes ineffective as a load carrying element only within the region δ, and the applied load is therefore being carried by the remaining unbroken fibres. The outcome of this approach is a model that fails within a single cross sectional layer. This approach seems to be appropriate for characterizing the strength of strands which do in fact fail within a single cross sectional layer. Once a strand is broken, it becomes totally detached from the rest of the specimen and is therefore ineffective as a load carrying element. From the description of a strand it is clear that its total length (which is equal to the specimen's length) becomes ineffective whereas in Rosen's analysis [4] only a small portion of a broken fibre is ineffective.

If we consider now the specimen to be composed of many strands connected in parallel, we have a situation analogous to that considered by Rosen [4] (i.e., a bundle of many fibres of length δ connected in parallel). The strand of length l, whose strength is characterized by Equations 8 and 9, replaces the fibre link of length δ whose strength is defined by $f(\sigma)$ and $F(\sigma)$ (Equation 2). Daniels' theory [10] is applicable for calculating the distribution of the average strand stress at specimen's failure, and the analysis of the specimen's strength is analogous to that of a bundle (Equations 3 to 7). Thus, the relative number of unbroken strands at strand stress σ_s is $1 - \Lambda(\sigma_s)$ where $\Lambda(\sigma_s)$ is defined in Equation 9. If the stress applied to the specimen is denoted by $\tilde{\sigma}_c$, it is related to the strand stress by:

$$\tilde{\sigma}_c = \sigma_s[1 - \Lambda(\sigma_s)] \tag{10}$$

The maximum strand stress at specimen failure is denoted by σ_{su} and is found by maximizing $\tilde{\sigma}_c$ in Equation 10,

$$\frac{d}{d\sigma_s}\{\sigma_s[1 - \Lambda(\sigma_s)]\}_{\sigma_s = \sigma_{su}} = 0 \tag{11}$$

which results in

$$\sigma_{su} = \frac{1 - \Lambda(\sigma_{su})}{\lambda(\sigma_{su})} \tag{12}$$

or, after substitution from Equations 8 and 9 and carrying out the integration, Equation 12 takes the form

$$\sigma_{su} = \frac{1 - \Omega(\sigma_{su})}{n\omega(\sigma_{su})} \tag{13}$$

The statistical mode of the composite strength is defined as the most expected failure stress of the specimen, $\bar{\sigma}_c$, and is obtained by substituting of σ_{su} from Equation 13 in Equation 10,

$$\bar{\sigma}_c = \sigma_{su}[1 - \Lambda(\sigma_{su})] \tag{14}$$

It is more convenient to rewrite $\bar{\sigma}_c$ in terms of the function Ω rather than Λ and this is done using the relation

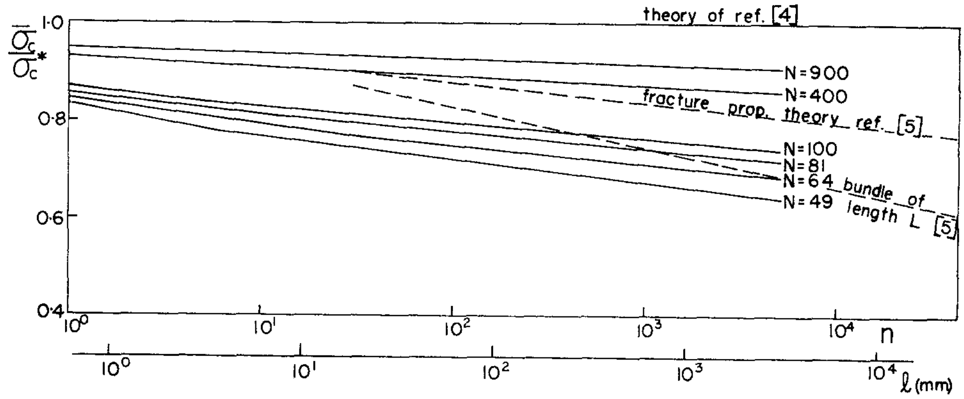

Figure 4 Variation of tensile strength with specimen's length for different strand size glass/epoxy composite.

$$1 - \Lambda(\sigma_{su}) = [1 - \Omega(\sigma_{su})]^n \qquad (15)$$

which is obtained from Equations 8, 12 and 13. Thus, the statistical mode of the composite strength is

$$\bar{\sigma}_c = \sigma_{su}[1 - \Omega(\sigma_{su})]^n \qquad (16)$$

In order to determine $\bar{\sigma}_c$ in Equation 16, σ_{su} has to be obtained first by solving Equation 13. This equation can be solved by introducing a standardized variable

$$Z = \frac{\sigma_B - \bar{\sigma}_B}{\psi_B} \qquad (17)$$

and the normal cumulative distribution function associated with this variable

$$\phi(Z) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{Z} \exp(-Z^2/2)\,dZ \qquad (18)$$

The associated cumulative function $\Omega(\sigma_{su})$, Equation 5 is then given in terms of $\phi(Z)$

$$\Omega(\sigma_{su}) = \phi(Z_{su}) \qquad (19)$$

where

$$Z_{su} = \frac{\sigma_{su} - \bar{\sigma}_B}{\psi_B} \qquad (20)$$

Substitution of Equations 4, 19 and 20 in Equation 13 yields the following equation which can be solved numerically:

$$\phi(Z_{su}) + \left(\frac{\bar{\sigma}_B}{\psi_B} + Z_{su}\right)\left[\frac{n}{\sqrt{(2\pi)}} \exp(-Z_{su}^2/2)\right] = 1 \qquad \ldots\ldots (21)$$

Once Z_{su} and $\phi(Z_{su})$ are known, σ_{su} and $\Omega(\sigma_{su})$ can be obtained from Equations 19, 20, 6 and 7 after substituting the appropriate strength distribution function of the fibres. For fibres characterized by the Weibull distribution function [8] their strength is given by Rosen [4].

$$f(\sigma) = \alpha\beta\,\delta\,\sigma^{\beta-1} \exp(-\alpha\,\delta\,\sigma^{\beta}) \qquad (22)$$

and when Equation 21 is rewritten in terms of the fibre parameters it becomes

$$\phi(Z_{su}) + \left[\left(\frac{N}{e^{1/\beta}(1 - e^{-1/\beta})}\right)^{\frac{1}{2}} + Z_{su}\right] \frac{n}{\sqrt{(2\pi)}} \exp(-Z_{su}^2/2) = 1 \qquad (23)$$

For each appropriate value of Z_{su}, which defines strength of a specimen by Equations 16 and 20, there exist pairs of values of N and n that solve Equation 23. These values which are, respectively the number of fibres in a strand and the length of the specimen divided by the ineffective length, define the size of the specimen that is expected to fail under the applied load. By assigning various values to Z_{su}, the strength of different size specimens is obtained. The result of this procedure is shown graphically in Fig. 4 and the details of the calculation are given in the Appendix.

5. Results and discussion

The object of the present study was to establish a failure model for unidirectional glass reinforced polymer when tensile load is applied in the fibre direction. In particular, the discrepancy between predicted failure geometry and the actual shape of a fractured specimen (Fig. 1) has been investigated. As a result a failure model has been developed, that does predict a failure pattern which is compatible with experimentally observed failure patterns.

Fig. 4 shows the variation of strength with specimen's length for specimens of different strand sizes. The failure stress $\bar{\sigma}_c$ is normalized with respect to Rosen's prediction σ_c^* [4], and the specimen's length is given either in terms of the number of ineffective lengths n (n = specimen's length/ineffective length), or as the real length of a specific E-glass/epoxy composite that was used in our tests. (E-glass with filament diameter of 0.013 mm and epoxy ERL-2256 and ZZL-0820 manufactured by Union Carbide).

The strength of a specimen, having a finite number of fibres (N) in each strand, is shown to decrease as the length increases. The magnitude of this decrease in strength value is larger for specimens having small strands than for specimens having large strands. In the limit, when N approaches infinity, our theory coincides with Rosen's [4] prediction of strength. For each specimen's length the present analysis predicts lower strength value than in [4], thus improving agreement between the analysis and experimental results. Zweben [5], who has proposed two modes of composite tensile failure, predicted similar results to our present results although by using different models. His results for the two failure modes are shown also in Fig. 4 by the dotted lines.

A direct verification of the size effect is very difficult and requires specialized equipment for preparing very large specimens. An opportunity for conducting tensile tests on long specimens was found by McKee and Sines [7] during a program to station meteorological instruments at very high altitude using a tethered balloon. The cable was made by drawing single-end S-glass yarn from 150 spools and impregnating them with epoxy resin. McKee and Sines report that specimens of three lengths have been tested, namely, 6, 30, and 120 in. The fibre diameter was 0.0004 in. and the total number of fibres in a cross section was 31 200. The experimental results concerning variation of tensile strength with specimen's length are not clearly stated in this report. Experimental strength values for two specimen's lengths are given in one of the figures (Fig. 3 of ref. [7]) as function of specimen's size. The lengths of the specimens for which the experimental results are plotted are not stated precisely, and the sizes of the specimens are given in terms of the number of "volume units". By drawing a line through those two points and measuring (from the graph) the fracture stresses for specimens of 10^7 and 10^9 volume units (corresponding to 1.8 and 180 in. long specimens), the values are 547 and 452 ksi respectively. This means that when the length of the specimen is increased by two orders of magnitude, from 1.8 to 180 in., it loses about 17.4% of its strength that it had at the shorter length. Now examine in Fig. 4 the line corresponding to $N = 49$, which is the predicted failure stress for the S-glass/epoxy specimens, and take the strength values for 1.8 and 180 in. long specimens. These values are, respectively, $0.730\sigma_c^*$, and $0.634\sigma_c^*$, indicating that the reduction of predicted strength for the longer specimen with respect to the shorter one is about 13.2%. The proximity of the experimental results to our prediction is not taken as a solid proof of the validity of the present theory on a quantitative basis. Many more experimental results as well as refinement of the theory are needed before being able to make such a statement. The main reason for bringing these experimental results is to demonstrate experimentally that the trend of size effect in the composite material discussed here is of the same order as predicted by our theory.

The failure geometry shown in Fig. 1 is typical to all the test specimens used in this programme. All specimens disintegrated upon failure into many strands, and there was a distinct difference between the E-glass and the S-glass reinforced specimens. The strand size in the E-glass specimens was larger than the predicted size, while the S-glass specimens seemed to have a better fit with the predicted strand size. Variation of strand size within each group of specimens was not too large and it was possible to determine by observation whether a fractured specimen contained E-glass or S-glass fibres.

Two typical strands taken from fractured E-glass and S-glass reinforced specimens are shown in Fig. 5. Attention should be given to two facts: (a) the larger strand size of the E-glass compared to the S-glass strand, and (b) the longer pull-out length of S-glass fibres compared to the E-glass specimen. Without referring to the exact strand size or the pull-out length that have been predicted (see appendix) by the present simplified analysis, the experimental results in Fig. 5 indicate that the present theory is at least in a quantitative agreement with experimental observations.

It is believed that the present model is a step forward in the understanding of failure mechanisms of unidirectional fibrous composites. The agreement between experimental results and the present analysis concerning failure geometry, strength values and strand size, is a good indication that the proposed failure model is a realistic one.

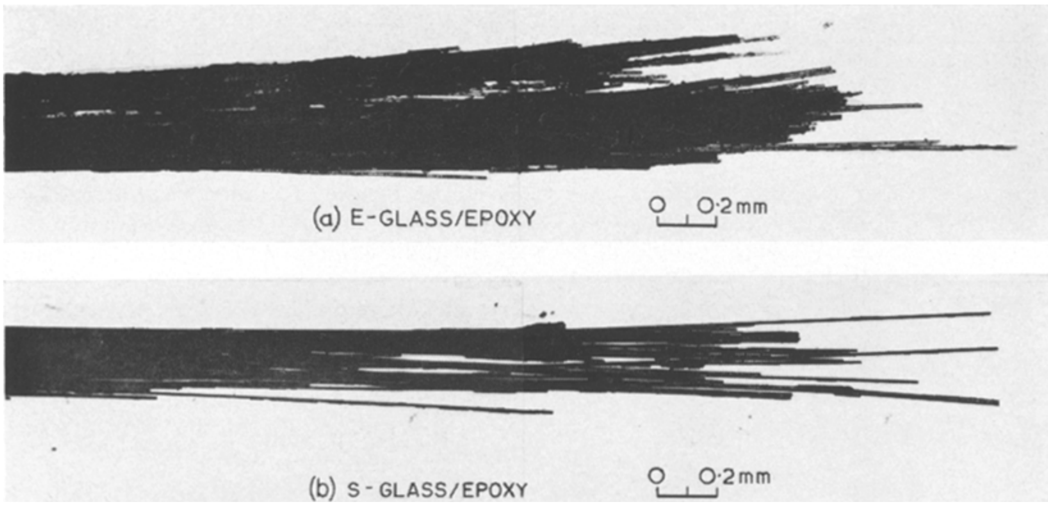

Figure 5 Typical fractured strand of (a) E-glass/epoxy, and (b) S-glass/epoxy composite material.

Appendix

A. Sample calculation of new model's strength (Equation 16)

It has been shown in the test that the longitudinal strength of the new model is given by $\bar{\sigma}_c$ in Equation 16. The solution of Equation 16 requires some simple numerical calculations, a sample of which is presented here. For fibres characterized by the Weibull distribution function their strength is given by:

$$f(\sigma) = \alpha\beta\delta\sigma^{\beta-1} \exp(-\alpha\delta\sigma^{\beta}) \quad \text{(A.1)}$$

and the associated cumulative distribution function:

$$F(\sigma) = 1 - \exp(-\alpha\delta\sigma^{\beta}) \quad \text{(A.2)}$$

The maximum fibre stress at bundle failure σ_f is obtained by maximizing $\tilde{\sigma}_B$ in Equation 3, thus

$$\sigma_f = (\alpha\beta\delta)^{-1/\beta} \quad \text{(A.3)}$$

When these values are substituted in Equation 7, the standard deviation ψ_B becomes

$$\psi_B = (\alpha\beta\delta)^{-1/\beta} N^{-\frac{1}{2}} [(1 - e^{-1/\beta}) e^{-1/\beta}]^{\frac{1}{2}} \quad \text{(A.4)}$$

The expectation of bundle (strand) strength is

$$\bar{\sigma}_B = (\alpha\beta\delta e)^{-1/\beta} \quad \text{(A.5)}$$

which is equal to the strength of the composite given by Rosen (Equation 1).

Now we have to solve Equation 23, which upon substituting the value $\beta = 4$, becomes

$$(1.875\sqrt{N} + Z_{su}) \frac{n}{\sqrt{(2\pi)}} \exp(-Z_{su}^{2}/2) = 1 - \phi(Z_{su}) \quad \text{(A.6)}$$

By selecting appropriate values for Z_{su} and using integral tables for $\phi(Z_{su})$, a relation between n and N is obtained for each value of Z_{su}. From Equation A.6 it is obvious that Z_{su} must be larger than $-1.875\sqrt{N}$. For the physical problem at hand since N and n are both positive numbers greater than one, it is concluded that Z_{su} must be a negative number. Thus, we have a permissible range of variation for Z_{su}.

The strand stress at specimen failure, σ_{su}, is obtained by substitution of Equations A.4 and A.5 in Equation 20:

$$\sigma_{su} = \bar{\sigma}_B \left\{1 + \frac{Z_{su}}{\sqrt{N}} [(1 - e^{-1/\beta}) e^{+1/\beta}]^{\frac{1}{2}}\right\} \quad \text{(A.7)}$$

and for $\beta = 4$ it becomes

$$\sigma_{su} = \bar{\sigma}_B \left[1 + 0.532 \frac{Z_{su}}{\sqrt{N}}\right] \quad \text{(A.8)}$$

Substitution of Equations A.8 and 19 in Equation 16 yields the final expression for calculating the strength:

$$\bar{\sigma}_c = \bar{\sigma}_B \left[1 + 0.532 \frac{Z_{su}}{\sqrt{N}}\right] [1 - \phi(Z_{su})]^{n} \quad \text{(A.9)}$$

Some numerical results obtained from Equations A.6 and A.9 are given in the following Table. The specimen length l is for E-glass/epoxy composite whose ineffective length is $\delta = 0.62$ mm.

B. Load transfer and strand size

When a unidirectional fibrous composite is stressed in the direction parallel to the fibres,

Z_{su}	$1-\phi(Z_{su})$	N	n	l (mm)	$\bar{\sigma}_c/\sigma_c{}^*$
− 4.5	0.999 996 6	49	7310	4530	0.633
		64	6000	3720	0.679
		81	5080	3150	0.715
		100	4420	2740	0.745
		400	1910	1185	0.872
		900	1220	756	0.914
− 3.8	0.999 927 6	49	369	229	0.691
		64	306	190	0.731
		81	263	163	0.761
		100	230	143	0.784
		400	102	63	0.892
		900	66	41	0.928
− 3.2	0.999 312 9	49	42	26	0.735
		64	36	22	0.768
		81	31	19	0.794
		100	27	17	0.815
		400	12	7	0.908
		900	8	5	0.938
− 2.6	0.995 339 0	49	7.0	4.3	0.78
		64	5.9	3.7	0.81
		81	5.1	3.2	0.83
		100	4.5	2.8	0.84
		400	2.1	1.3	0.92
		900	1.4	0.9	0.95
− 1.8	0.964 070	49	1.08	0.67	0.83
		64	0.93	0.58	0.85
		81	0.81	0.50	0.87
		100	0.72	0.45	0.88
		400	0.34	0.21	0.94
		900	0.22	0.14	0.96

stress concentrations are built up in the vicinity of fibre breaks during loading. The stress field in the vicinity of fibre breaks has been studied by many investigators, using various idealized models. A short account of this work can be found in [11].

Three analytical approaches have been followed in studying the details of the stress distribution near fibre breaks. In the first, [12, 13, 4] the matrix is assume to be an elastic material, and the maximum stress induced in it is still below the yield stress (or fracture strength) of the matrix. A second approach which is suitable for metallic matrices that can undergo plastic flow, has been presented by Kelly and Tyson [14] where they assume that the applied stress is of sufficient magnitude to result in plastic flow in the matrix material near the fibre ends. A third approach has been proposed by Outwater [15] for a large volume fraction of discontinuous brittle fibres in a polymer matrix.

In the case of a brittle polymer matrix all the predictions of stress field near fibre ends in an elastic matrix have shown that the maximum shear stress in the matrix, at practical load levels, is far above the fracture stress of the resin. For example, the predicted maximum shear stress in an epoxy matrix containing 60% volume fraction of E-glass fibres, is approximately 0.3 times the average stress applied to the composite (based on [4]). The measured failure shear stress of the same epoxy is about 0.05 times the failure stress of the same composite. Failure of the resin (or the bond between fibre and resin) near the fibre end must therefore have occurred at rather small loads. Outwater [15] suggests that the shrinkage of the resin on curing subjects the fibres to a radial pressure p which is equal to that exerted by a thin walled tube of thickness t, internal diameter d and hoop stress σ_ϕ

$$p = \frac{2\sigma_\phi t}{d} \tag{B.1}$$

where t is half the separation between the surfaces of adjacent fibres, d is fibre diameter and σ_ϕ is taken to be yield stress of the resin in tension. When debonding occurs near the fibre end, the load can be transferred to the broken fibre by the frictional force that exists between the fibre and the matrix.

Neglecting shear deformations in the resin, the stress build-up in the fibre (σ_f) is linear

$$\sigma_f = \frac{4\mu p x}{d} \tag{B.2}$$

where μ is the coefficient of friction between fibre and matrix and x is the distance along the fibre measured from its broken end. The distance δ, required to build the fibre stress to its undisturbed value is obtained by substituting in Equation B.2 the value of the undisturbed fibre stress $\sigma_f = \sigma_c/v_f$, where σ_c is the stress applied to the composite and v_f the fibre volume fraction. Thus, from Equations B.2 and B.1

$$\delta = \frac{\sigma_c\, d^2}{8\mu\, \sigma_\phi\, t v_f} \tag{B.3}$$

The value of μ depends on the finish applied to the glass, and the value of σ_ϕ depends on the time after curing of the resin. Taking $\mu = 0.4$ [16], and using experimental results [17] from stress relaxation experiments with epoxy resin, an estimated value of σ_ϕ is taken to be 7.50 kg/mm². Substitution of these values in Equation B.3 for $v_f = 0.6$, $d = 0.013$ mm and $t = 0.0019$ mm results in

$$\delta = 0.0062\sigma_c$$

and for composite strength value of 100 kg/mm^2 the maximum value of δ is:

$$\delta_{max} = 0.62 \text{ mm}$$

or:

$$\delta_{max} \simeq 48d$$

This value of δ is only an estimate of the length over which debonding occurs near fibre breaks. We shall refer to this value as the "ineffective length" in the subsequent statistical analysis of a strand's strength.

When a few neighbouring fibres fail at a single cross sectional layer whose thickness is about δ, they form a strand of zero stress at the broken end. The load that has to be transferred now by the matrix surrounding the strand is considerably larger than the load that was transferred after failure of a single fibre. The larger the strand size, the longer is the length that is required to build the stress in the strand to its undisturbed value. Since the test specimens are of finite length, the size of the strand is finite too.

The strand is assumed to have a circular cross section of diameter D, surrounded by a thin walled matrix tube of thickness t, through which the load is transferred to the strand by means of friction. Equation B.1 still holds if d is replaced by D, and Equation B.3 becomes:

$$\delta_{strand} = \frac{\sigma_c D^2}{8\mu \sigma_\phi t} \qquad \text{(B.4)}$$

The maximum value of δ_{strand} varies from a few mm to almost the full specimen's length, depending on the location of the fractured end of the strand. We shall assume a value which is half the specimen's length. The value of the applied stress σ_c is taken to be 90 kg/mm^2, which is close to the composite strength, since most of the fibres fail at stress levels close to the composite strength. Upon substituting these values in Equation B.4 it is found that the strands' diameter of E-glass/epoxy 100 mm long specimen is:

$$D \simeq 0.16 \text{ mm}$$

and the number of fibres (N) in the strand is:

$$N = v_f \left(\frac{D}{d}\right)^2 \simeq 90$$

It is worth noting that for S-glass/epoxy specimens having the same fibre volume content $(\sigma_c)_{max}$ is about twice the value for E-glass/epoxy and the strand size is therefore smaller, having a diameter $D \simeq 0.113$ mm and $N = 45$ fibres per strand. This change of strand size has been observed and S-glass/epoxy specimens fractured into strands of smaller diameters than the E-glass /epoxy specimens (Fig. 5).

Acknowledgements

This research has been sponsored in part by the Air Force Materials Laboratory (MANE) Wright-Patterson AFB, Ohio 45433 through the European Office of Aerospace Research, OAR, United States Air Force under contract F61052-68-C-0015.

References

1. R. W. JECH, D. L. MCDANIELS, and J. W. WEETON, Fiber Reinforced Metallic Composites, Proc. 6th Sagamore Ordnance Materials Research Conf. (August 1959), 116.
2. A. KELLY and G. J. DAVIES, *Met. Rev.* **10** (1965) 37.
3. N. J. PARROTT, Rubber Plastics Age (March 1960) 263.
4. B. W. ROSEN, *AIAA J.*, **2** (1964) 1985.
5. C. ZWEBEN, *AIAA J.*, **6** (1968) 2325.
6. A. G. METCALFE and G. K. SCHMITZ, Effect of Length on the Strength of Glass Fibers, ASTM Preprint No. 87 (June 1964).
7. R. B. MCKEE, JUN. and G. SINES, A Statistical Model for the Tensile Fracture of Parallel Fiber Composites, ASME Winter Annual Meeting, New York (December 1968).
8. W. WEIBULL, *J. Appl. Mech.* **18** (1951) 293.
9. J. M. LIFSHITZ and A. ROTEM, Technicon IIT, Israel, MED Rept. 25 (April 1970).
10. H. E. DANIELS, *Proc. Roy. Soc. London* **183A** (1945) 405.
11. B. W. ROSEN and E. FRIEDMAN, "Mechanics of Whisker-Reinforced Composites, Whisker Technology", Ed. by A. P. Levitt (J. Wiley and Sons, Inc, New York, 1970).
12. H. L. COX, *Brit. J. Appl. Phys.* **3** (1952) 72.
13. N. F. DOW, Study of Stress, Near a Discontinuity in a Filament-Reinforced Composite Material, G. E. Comp. Space Sciences Lab, Space Mechanics Memo No. 102 (January 1961).
14. A. KELLY and W. R. TYSON, "Fiber-strengthened Materials, High Strength Materials," Ed. by V. F. Zackay, (John Wiley and Sons Inc, New York, 1965).
15. J. O. OUTWATER JUN, *Mod. Plast.* **33** (1956) 156.
16. A. KELLY, "Strong Solids" (Clarendon Press, Oxford, 1966) 139.
17. J. M. LIFSHITZ, Specimen Preparation and Preliminary Results in the Study of Mechanical Properties of Fiber Reinforced Material, Tech. Rpt. AFML-69-89 Part I (July 1969).

Received 6 December 1971 and accepted 8 February 1972.